Localization of firms in conditions of triangular distribution of consumers and uncertain quality of firms

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A R T I C L E　I N F O

A B S T R A C T

One of the main problems of localization models is simplifying assumptions on consumers' distribution on the streets or city. Localization methods usually use uniform distribution patterns while in real world, this kind of distribution is unavailable, and the consumers are mostly accumulated in centers rather than margins. Using triangular distribution for consumers and concerning uncertainty of the quality of products of firms, this paper deals with localization of firms. The results indicated that if the quality of products differs highly, then just one firm will sell its products in the market. If this difference is partial and negligible, then both firms will be active in the market. If either firm 1 or firm 2 is the only seller in the market, then it gets the highest price at final points. Furthermore, if the indifferent consumer is at the left side of the center, firm 1 will get the highest price at left margin (point 0) and if the indifferent consumer is at the right side of the center, then firm 1 will get the highest price at right side (point 1) and in both conditions, firm 2 will get the highest price at the center.

1. Introduction

Localization is one of the key steps in building firms, the results of which have a huge impact on the future of the firm and the region. Deciding on the location and conditions of the creation or development of an industrial unit is also among the most fundamental decisions of investors. These decisions could play a crucial role in strategic direction of the firm and ensure its profitability in the long run. Thus, if there is not proper investigation regarding the chosen place, it could affect the firm's life in the long run and even lead to its closure.

Nowadays, the globalization of activities, the rapid growth of awareness and knowledge of the people, the widespread expansion of markets, the speed of dissemination of market and product information, and innovations in knowledge and technology have provided economic activities with a lot of limitations. In addition, it is now necessary that the factories pay attention to the competition issue while having access to competitive advantages and customer-oriented categories. Competitive location models were introduced by Hotelling through *game theory* approach concerning the competition between two ice-cream vendors in 1929. Next studies are for the improvement of one or several assumptions of Hotelling model and taking them into consideration in a more general level. The dramatic changes in the last decades regarding the patterns of competitive location, the ever-increasing rise of these changes,

and competitive environments have forced firms to compete with other competitors in order to survive and increase their market share. In this regard, the use of proper and accurate localization tools and methods is actually effective in realizing the goals of the firms (Bagherinejad & Niknam, 2012).

Since making establishment policies of industries without knowledge and awareness leads to decrease or disappearance of the efficiency of economic system, the significance of location study becomes clear (Vinay & chakra, 2005). Place could be interpreted in two ways: the physical location of a particular consumer or the distance between the desired attributes of a trade mark with a particular consumer with the characteristics of the commercially-marketed one that he/she has actually bought (Shy, 1995). Choosing the optimal location for the establishment of a firm or production organization plays a key role in its strategic decisions, so it could guarantee long-run profitability of the firm and, in the absence of these studies, endangers the life of the firm or institution (Macoei et al., 2014).

What follows are some studies regarding the choice of firm locations. Darling (2001) focused on the importance of competitive locating for access to the European consumer market while introducing a model for helping executives marketing to find the optimal location in the European market. Hosun et al (2003) showed that the best place to build a factory is the one being in a better position in terms of concessional facilities. In a linear city model with a uniform distribution of consumers, Christou & Vettas (2005) considered two firms that sought to maximize their expected profits in uncertain terms. Their results indicated that the equilibrium position of the firms depends on the expected quality and the power of horizontal differentiation.

Chen and Riordan (2007) considered the spokes model which was the generalized model of the circular city. They numbered N streets with a length equal to ½ in which on all streets, consumers were evenly distributed with 2/N consumers on each spoke (streets) and proved that the equilibrium price is higher than the final cost even when the number of firms is high. Granot et al (2010) analyzed the Hotelling model, in which the space of a linear city is in a competitive and exclusive environment. They used the Nash equilibrium concepts to show the optimal location for firms seeking maximum profits. Lijesen and Reggiani (2013) raised two questions and resolved them in order to expand Chen and Riordan spokes model: First, the location of the firms and, secondly, how parts of the market that are not covered by the firms so far are affecting the decisions of the firms operating in the market. They analyzed the location of the place and the cost of transportation from the linear state in Chen and Riordan model in a square order.

Kieron et al (2014) analyzed the optimal social location in a monopolistic model in the uncoordinated distribution of consumers. They calculated the welfare gains from regulating the location of firms and showed how this prosperity would differ with the distribution of consumers. While the regulation of the firm location in symmetric distributions is sufficient in order to maximize the welfare of consumers, in asymmetric distributions, price regulation is also necessary to provide optimal social welfare.

Naveen et al (2016) conducted a survey on location choice with a new approach and stated that factors affecting the location could be categorized into two groups. They analyzed 151 articles published in international trade and management journals since 1975 and reached the results. They suggested that this approach could improve further research on localization by providing a descriptive model.

Shahbazi and Salimian (2017) studied the optimal location of firms inspired by the Prick Lijesen and Reggiani's spokes model (2013) while changing the distribution of consumers from uniform distribution to triangular distribution in a two-stage game. One of the most important results was that with increasing distance between firms, the price of the market rises significantly indicating that as far as firms are farther away, competition between them in the product market declines and leads to higher prices. Besides, if the distance between the two firms is the same as the center of the city, then both firms will get the same share of the market and will be willing to choose a location close to the center (minimal differentiation).

Based on the original Hotelling model, Shahbazi and Salimian (2018) took into account the two types of experienced and inexperienced consumers to locate firms in these situations. One of the most important results was that if two firms are located at a point or at a distance from the center, due to differences in consumer tastes, they will not demand the same equilibrium prices while with increasing transportation costs, high-quality consumer goods manufacturing firm will be closer to the center and the low-quality goods producer will be away from the center.

One of the most important discussions is that the simplification assumptions have always been used in localization models. One of these models being the basis of other localization models which always has its effect on other location-related tasks is the Hotelling model. Most of the next location research is to improve one or more hypotheses of the Hotelling model and make it more general. The most important point in these generalizations is that competitive localization models are inherently unstable and with the slightest change in a premise or a parameter, totally different results will be achieved (Macoei et al

, 2014). Melaniphy (1999) investigated early return businesses and found that over 50 percent of them in the first year and about 30 percent of them in the second year become bankrupt and take on another job. Although all aspects of the provision of services are investigated at the beginning of the establishment of these businesses, neglecting the important issue of the location causes the unit to not achieve the desired profit and goals. Therefore, the problem (optimal localization) matters so much and it is necessary to use more realistic assumptions and parameters so that the firm could execute the correct and optimal decisions and achieve the desired profitability.

This article is organized in five sections. After the introduction, the triangular distribution model is introduced in the second section. The third section deals with the model used in this study which is concerned with two equilibrium price and equilibrium location subdivisions. In the fourth and final sections, conclusions and suggestions are presented.

2. The triangular distribution model

X random variable has triangular distribution and takes the values in S= [a, b]. Here, a is the starting point of the streets and b is their end point, and it is assumed that a point like c is the connection center of these streets. The probability of consumers' distribution in sub-interval [a,c] increases linearly. It means the more we become closer to center, the more the number of consumers located on that street increases. Furthermore, in sub-interval [c, b], the probability of distribution of consumers decreases linearly and by becoming far from the center, the number of consumers on that street decreases. Thus, the density function of this variable has triangular form. The triangular distribution is denoted by Tria (a, c, b) and its density function is obtained as:

$$PDF = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a \le x < c \\ \frac{2}{b-a} & x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & c < x \le b \end{cases}$$

(Shahbazi & Salimian, 2017).

In Christou and Vettas (2005), a unit-length street is considered which could be considered as two streets with one center simply by placing a center for it at point 1/2. However, instead of uniform distribution of consumers on every street, in this case, consumers in every street are distributed triangularly. By putting the values, b = 1 a = 0, c = (1) / 2, the triangular density function is as follows:

$$= \begin{cases} 4x & 0 \le x < \frac{1}{2} \\ 4 - 4x & \frac{1}{2} \le x \le 1 \end{cases}$$

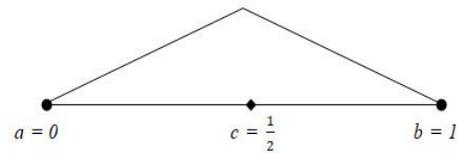The figure 1 shows the function of the density of the triangular distribution:

Figure 1: The shape of the triangular distribution density function

In this case, consumers are assumed to be distributed at zero and one distance and the middle point (Mid), which includes more consumers, is the same as c (street center) (Shahbazi and Salimian, 2017).

3. Model

Christou and Vettas (2005) model is used in this paper. However, the distribution of consumers is not uniform and has a triangular distribution function. Christou and Vettas considered the Hotelling linear city in the interval [0, 1] assuming that there are two firms on this street each of which produces only one type of product and in the interval [0 , 1]. In addition, consumers throughout the street are distributed uniformly. They also considered transport costs as a quadratic function. Following Christou and Vettas (2005), it is assumed that a consumer situated at point won the interval [0 , 1] purchases one unit of product from the firm i= 1, 2, and earns a surplus of:

$$u(w, i) = R - t(w - y_i)^2 + q_i - p_i \qquad (1)$$

where R is the reserve value of the product that is assumed to be high enough so that all consumers buy one unit of the product. In other words, the market is fully covered. Besides, t is the cost of transportation per unit, y_i is the location of firm i, q_i is the product quality (assuming that the difference in the quality of the firm's products is unknown for consumers) and p_i is the price of the product i. This function shows that product differentiation is both horizontal and vertical. In order to obtain the effect of the uncertainty of quality on the location of the firms, it is assumed that the quality of product i, q_i, is a random value, which is unknown when choosing the location of the firms. In this situation, the game will be as follows:

1. Firms simultaneously choose their products.
2. The difference in the quality of firms (q_i-q_j) is evident and common knowledge.
3. Firms simultaneously choose the price of their products.
4. By looking at the location of the firms, the price and quality of the products, each consumer purchases a unit from one of the firms.

Each firm seeks to maximize its expected profit. On the other hand, every consumer also seeks to

maximize its net surplus of purchases. This suggests that the change of location for firms is very costly after identifying the product quality for consumers (Christou and Vettas, 2005). It is assumed that the difference in the quality of the firms (q_i-q_j) which is random is in H_1, S, H_2, H_1, and means that the difference in the quality of the products of the two firms is high and the quality of the firm 1 is better than the quality of firm 2. S indicates that the difference in the quality of the products of the two firms is minor and consumers do not prefer one firm's product to another firm's product. Furthermore, H2 means that the difference in the quality of the products of the two firms is huge and the quality of the firm 2 is much better than the quality of the firm 1. Suppose the difference in the quality of the two firms is uniform and distribute over the interval $[-\frac{1}{2}, \frac{1}{2}]$. In addition, suppose that the difference of two firm's product quality in $[-\frac{1}{2}, -\frac{1}{4}]$ is equivalent to H_1, in $[-\frac{1}{4}, \frac{1}{4}]$ is equivalent to S, and eventually within $[\frac{1}{4}, \frac{1}{2}]$ is equivalent to H_2.

As the quality difference is a random value, the profit allocated to each firm will be a random value for each location. In addition, for simplicity, it is assumed that the marginal cost of production of each unit in all stages is zero.

Now, we use the backward induction method to obtain the equilibrium value first.

3.1. Equilibrium Price

Since each of the firm 1 and firm 2 could select any location on the line, so for simplicity we assume that firm 1 is located on the left side of firm 2 ($y_1 \leq y_2$). Now, with the location of the firms, the function of the demand of each firm will be obtained. Suppose z is the demand for firm 1 and therefore 1-z will demand for firm 2 ($z \in (0, 1)$). The demand for two firms will be the same as obtaining a consumer place without buying any two products from firms 1 and 2. Thus, having the relation 1 and the fact that the consumer is located indifferently at the point z, we have:

$$p_1 + t(z - y_1)^2 = -q + p_2 + t(z - y_2)^2 \qquad (2)$$

Where $q = q_2 - q_1$. It is clear that depending on the quality level of the product produced by both firms, q can be positive, negative, or zero. The location of the indifferent consumer depends on the location and price of the existing firms. Besides, this location depends on the transportation costs and the quality difference of the two products, so $z = z$ (p_1, p_2, y_1, y_2, q, t). By solving Eq. 2 and for $z \in (0,1)$ we have

$$z^* = \frac{y_1 + y_2}{2} + \frac{p_2 - q - p_1}{2t(y_2 - y_1)}$$

Therefore, the profit function of the firms will be as follows:

$$\pi_1 = p_1 z \quad , \quad \pi_2 = p_2(1 - z) \qquad (3)$$

So:

$$z = \begin{cases} 0 & if \\ \quad \frac{y_1 + y_2}{2} + \frac{p_2 - q - p_1}{2t(y_2 - y_1)} \leq 0 \\ 4\left(\frac{y_1 + y_2}{2} + \frac{p_2 - q - p_1}{2t(y_2 - y_1)}\right) & if \\ \quad 0 < \frac{y_1 + y_2}{2} + \frac{p_2 - q - p_1}{2t(y_2 - y_1)} < \frac{1}{2} \\ 4 - 4\left(\frac{y_1 + y_2}{2} + \frac{p_2 - q - p_1}{2t(y_2 - y_1)}\right) & if \\ \quad \frac{1}{2} \leq \frac{y_1 + y_2}{2} + \frac{p_2 - q - p_1}{2t(y_2 - y_1)} < 1 \\ 1 & if \\ \quad \frac{y_1 + y_2}{2} + \frac{p_2 - q - p_1}{2t(y_2 - y_1)} \geq 1 \end{cases} \qquad (4)$$

These functions are derived from the triangular distribution of consumers. Now, we derive equilibrium prices:

Theorem 1: In terms of triangular distribution and uncertainty of quality, the equilibrium prices of firms 1 and 2 for $0 \leq y_1 \leq y_2 \leq 1$ are:

$$p_1^* = \begin{cases} t[(y_2 - 1)^2 - (y_1 - 1)^2] - q & if \\ \quad q < -\frac{1}{4} \\ -\frac{ty_1^2 + 2ty_1 - ty_2^2 - 2ty_2 + q}{3} & if \\ \quad q \in \left[-\frac{1}{4}, \frac{1}{4}\right] \;, \quad 0 < z < \frac{1}{2} \\ -\frac{ty_1^2 - 4ty_1 - ty_2^2 + 4ty_2 + q}{3} & if \\ \quad q \in \left[-\frac{1}{4}, \frac{1}{4}\right] \;, \quad \frac{1}{2} \leq z < 1 \\ 0 & if \qquad q > \frac{1}{4} \end{cases} \qquad (5)$$

$$p_2^* = \begin{cases} 0 & if \qquad q < -\frac{1}{4} \\ \frac{ty_1^2 - 4ty_1 - ty_2^2 + 4ty_2 + q}{3} & if \\ \quad q \in \left[-\frac{1}{4}, \frac{1}{4}\right] \;, \quad 0 < z < \frac{1}{2} \\ \frac{ty_1^2 + 2ty_1 - ty_2^2 - 2ty_2 + q}{3} & if \\ \quad q \in \left[-\frac{1}{4}, \frac{1}{4}\right] \;, \quad \frac{1}{2} \leq z < 1 \\ q - t(y_2^2 - y_1^2) & if \qquad q > \frac{1}{4} \end{cases} \qquad (6)$$

Proof: To prove this, it suffices to put Equation (4) in Equation (3). q <-1/4 indicates that the firm's quality is high enough and therefore firm 1 will be the only vendor on the market. q> 1.4 shows that the firm's quality is high enough and therefore firm 2 will be the only vendor on the market. Other scenarios are for cases where the product quality difference is minor and depends on the location of the firms, so

both firms have a positive sales value. If the product differentiation is only horizontal, then q = 0 and simply equilibrium prices can be obtained. Besides, if the product differentiation is only vertical, then $y_1=y_2$ and the equilibrium price equals the product quality difference (q). Bontems and Meunier (2002) gained almost the same results taking into account the uncertainty of quality and the fact that a higher-quality product manufacturer pays an additional cost of c. Considering the triangular distribution of consumers, Shahbazi and Salimian (2017) aimed at locating firms in these conditions. The equilibrium prices obtained by them (taking into account only two streets with one center) were approximately similar to these results noting that instead of the q parameter, they have considered the experienced consumer and disutility caused by consuming undesirable goods. However, they replaced the q parameter with the experienced consumer and the inadequate use of undesirable goods.

3.2. Equilibrium location

In this section, we are looking for the equilibrium of firm's location given equilibrium prices. Firms will be located where they can maximize their profit based on the location of the competing firm. The expected profit of firm i with $E\pi_i(y_1, y_2)$ is shown, which is based on the difference in the quality of the two firms. In this situation, the expected profit function of firm 1 will be as follows:

$$E\pi_1(y_1, y_2) = \int_{-\frac{1}{2}}^{-\frac{1}{4}} \pi_1^m(y_1, y_2)\, dF + \int_{-\frac{1}{4}}^{\frac{1}{4}} \pi_1^c(y_1, y_2)\, dF + \int_{\frac{1}{4}}^{\frac{1}{2}} \pi_1^c(y_1, y_2)\, dF \tag{7}$$

Where $F(y) = \frac{2y+1}{2}$ is the cumulative distribution function of the parameter q that is uniformly distributed over the interval $[-\frac{1}{2}, \frac{1}{2}]$. Moreover, in this case, due to the existence of a triangular distribution, $\pi_1^c(y_1, y_2)$ is considered in two modes ($\frac{1}{2} \le z < 1, 0 \le z < \frac{1}{2}$). It should be noted that depending on the difference in product differentiation, both types of corner and internal solutions may occur. $\pi_1^m(y_1, y_2)$ shows a state that only firm 1 will sell its products and $\pi_1^c(y_1, y_2)$ shows the profit function of firm 1, in which both firms 1 and 2 will sell their products. By placing the equilibrium prices in relations 5 and 6 in the profit function (Equation 3):

$$\pi_1^m(y_1, y_2) = t[(y_2 - 1)^2 - (y_1 - 1)^2] - q \tag{8}$$

$$\pi_1^c(y_1, y_2) = \begin{cases} \dfrac{(ty_1{}^2 + 2ty_1 - ty_2{}^2 - 2ty_2 + q)^2}{18t(y_2 - y_1)} & 0 < z < \dfrac{1}{2} \\[2mm] \dfrac{(ty_1{}^2 - 4ty_1 - ty_2{}^2 + 4ty_2 + q)^2}{18t(y_2 - y_1)} & \dfrac{1}{2} \le z < 1 \end{cases}$$

Similarly, the expected profit function of firm 2 will be as follows:

$$E\pi_2(y_1, y_2) = \int_{-\frac{1}{4}}^{\frac{1}{4}} \pi_2^c(y_1, y_2)\, dF + \int_{-\frac{1}{4}}^{\frac{1}{4}} \pi_2^c(y_1, y_2)\, dF + \int_{\frac{1}{4}}^{\frac{1}{2}} \pi_2^m(y_1, y_2)\, dF \tag{9}$$

In this case, in the presence of a triangular distribution, $\pi_1^c(y_1, y_2)$ is considered in two modes ($\frac{1}{2} \le z < 1, 0 < z < \frac{1}{2}$). Since a firm can be placed in the same location as the competitor, right or left of the firm, compared to the other firm, so the expected profit functions of firms 1 and 2 will be as follows:

$$\pi_1 = -\frac{56ty_1^2 - 80ty_1 - 56ty_2^2 + 80ty_2 - 9}{96} \tag{10}$$

$$\pi_2 = \frac{56ty_1^2 - 32ty_1 - 56ty_2^2 + 32ty_2 + 9}{96} \tag{11}$$

Now, we can present Theorem 2.

Theorem 2: The equilibrium location of firms 1 and 2 is symmetrical on both sides of the center and is $y_1 \le \frac{5}{7}$, $y_2 \le \frac{2}{7}$, and under these conditions the profit of both firms will be maximized at 0.

Proof: To prove this, it suffices to derive from relations 10 and 11 relative to the location of two firms 1 and 2 (y_1, y_2) and obtain the equilibrium position:

$$\frac{\partial \pi_1}{\partial y_1} = \frac{t(5 - 7y_1)}{6} \ge 0 \Leftrightarrow y_1 \le \frac{5}{7} \tag{12}$$

$$\frac{\partial \pi_2}{\partial y_2} = \frac{t(2 - 7y_2)}{6} \ge 0 \Leftrightarrow y_2 \le \frac{2}{7} \tag{13}$$

It is clear from the obtained relationships that the profit of both firms 1 and 2 will be maximized if they move towards point 0. These results show that if the differences in the quality of the two firms are sufficiently low, then the two firms will be deployed at a single point which will also be seen in the real world where homogeneous goods manufacturers gather in one place. Hotelling (1929), in his linear model, showed that homogeneous commodity producers are based in the city center.

Shahbazi and Salimian (2017) showed that in the placement of firms with a triangular distribution approach, the firm 1 could move its business toward firm 2 to gain a larger share of the market (the principle of minimal differentiation) as firms produce less differentiated products by moving to the center of products. They also showed that, if firm 1 approaches to firm 2, there will be no equilibrium and if it is exactly at the point where the firm 2 is located, its profit will be zero. Thus, it is best to go back to the left again because the price process is happening and there is no equilibrium. Consequently, firms could not be very close to each other (Shy, 1995). It should be noted that in the uncertainty of quality due to the difference (though small) between the products of the two firms, it will still not settle at a profit point.

In the model of different types of consumers (experienced and inexperienced), Shahbazi and Salimian (2018) also showed that the establishment of two firms in a symmetrical position relative to the center would not earn zero profits by firms due to the existence of experienced consumers. Obviously, if two firms are in exactly the same place, they will force the firms to battle which ultimately should receive this low price from experienced consumers (the profit will be zero here) and it is better that the firms again get away from each other.

Theorem 3: Firms 1 and 2 receive the same equilibrium prices whenever:

$$q = -t(y_1^2 - y_1 - y_2^2 + y_2)$$

Proof: To prove this, it is enough to relate equilibrium prices in both intervals ($\frac{1}{2} \le z < 1, 0 < z < \frac{1}{2}$):

$$-\frac{t{y_1}^2 + 2ty_1 - t{y_2}^2 - 2ty_2 + q}{3}$$
$$= \frac{t{y_1}^2 - 4ty_1 - t{y_2}^2 + 4ty_2 + q}{3}$$

$$-\frac{t{y_1}^2 - 4ty_1 - t{y_2}^2 + 4ty_2 + q}{3}$$
$$= \frac{t{y_1}^2 + 2ty_1 - t{y_2}^2 - 2ty_2 + q}{3}$$

In both cases, the equation will be established if $q = -t(y_1^2 - y_1 - y_2^2 + y_2)$.

Shahbazi and Salimian (2018) explored the localization of firms in the presence of two types of experienced and inexperienced consumers. They analyzed the Hotelling linear model by entering the second type of consumers, and showed that if two firms received the same price, they would be located at a distance from the center; which exactly corresponded to the results of the present research.

In their triangular distribution model, Shahbazi and Salimian (2017) showed that if the firms' positions were symmetrical, then the shares of both firms would be the same and both firms would demand the same prices. These results are also consistent with the results of the Hotelling model (1929). In an analysis of a market for product differentiation in terms of diversity, Nicholas (1989) evaluated the equilibrium and the optimality. The results showed that in a three-stage game in which firms enter the first stage, in the second stage of diversification and in the third stage they choose their prices, there is a perfect equilibrium of the game. In equilibrium, products are symmetric in their distributional characteristics, with the same prices being offered. Besides, Lijesen and Reggiani (2013, 2016) achieved similar results and showed that if the positions of the two firms are symmetric, then both firms will obtain the same share of the market and receive similar prices.

Theroem 4: If firms 1 and 2 are the only vendors on the market, then they receive the highest price at the margin points. In other words, if firm 1 is the only vendor of the product on the market, then at point 1, and if firm 2 is the only vendor of the product on the market, then at point 0 it will receive the maximum price and vice versa.

Proof: To prove this, we take the first derivative of relations 5 and 6 (in the area where both firms are monopoly) with respect to location of each firm and obtain the relations (suppose t=1):

$$\frac{\partial}{\partial y_1}(t[(y_2 - 1)^2 - (y_1 - 1)^2] - q) = 0$$

$$\frac{\partial}{\partial y_2}\left(q - t({y_2}^2 - {y_1}^2)\right) = 0$$

By solving these two equations, finally, the location where firms can get the maximum price are:

$$y_1 \le 1 \text{ , } y_2 \le 0 \qquad \text{or} \qquad y_1 \le 0 \text{ , } y_2 \le 1$$

Thus, two firms will be located in the final points. The place for firm 2 is determined to be located at point 0. Firm 1, if it is located at points prior to point 1 (closer to firm 2), should prepare itself for a price competition with firm 2.

Theorem 5: In a triangular distribution and a state of uncertainty of quality, if the indifferent consumer is in the interval $(0, \frac{1}{2})$, $(0 < z < \frac{1}{2})$, then firm 1 has the highest price in the margin (point 0), and firm 2 will receive the highest price at the center. Besides, if the indifferent consumer is in the interval $(\frac{1}{2}, 1)$, $(\frac{1}{2} < z < 1)$, then firm 1 receives the highest price on the margin (point 1), and the firm 2 receives the highest price at the center.

Proof: To prove this, it suffices for equations 5 and 6 (in the area where both firms sell their products) derive from the location of each firm and obtain the relations (suppose t=1):

$$\frac{\partial}{\partial y_1}\left(-\frac{t{y_1}^2 + 2ty_1 - t{y_2}^2 - 2ty_2 + q}{3}\right) = \frac{-2y_1 - 2}{3} \quad ,$$
$$0 < z < \frac{1}{2}$$

This phrase is always negative, so the firm 1 can get the highest price by moving to the margin (point 0). For firm 2, the situation is as follows:

$$\frac{\partial}{\partial y_2}\left(\frac{t{y_1}^2 - 4ty_1 - t{y_2}^2 + 4ty_2 + q}{3}\right) = \frac{-2y_2 + 4}{3} \quad , \quad 0$$
$$< z < \frac{1}{2}$$

This phrase is always positive, so the firm 2 can get the highest price by moving to the center. In the interval ($\frac{1}{2} \le z < 1$):

$$\frac{\partial}{\partial y_1}\left(-\frac{ty_1{}^2 - 4ty_1 - ty_2{}^2 + 4ty_2 + q}{3}\right) = \frac{-2y_1 + 4}{3} \quad ,$$

$$\frac{1}{2} \leq z < 1$$

This phrase is always positive, so the firm 1 can get the highest price by moving to the margin (point 1). For firm 2, the situation is as follows:

$$\frac{\partial}{\partial y_2}\left(\frac{ty_1{}^2 + 2ty_1 - ty_2{}^2 - 2ty_2 + q}{3}\right) = \frac{-2y_2 - 2}{3} \quad , \quad \frac{1}{2} \leq z < 1$$

This phrase is always negative, so the firm 2 can get the highest price by moving to the center.

Christo and Vettas (2005) also showed that if both firms sell their products on the market, then the optimal equilibrium location will be that either the two firms will be at one point or both cross borders with each other. The results of the models of experienced and inexperienced consumers (Shahbazi and Salimian, 2018) and the triangular distribution of Shahbazi and Salimian (2017) also confirm these results.

4. Conclusion and suggestions

Localization is among one of the most important factors affecting decision making of individuals and firms. Finding the right place to reduce competition between firms, reduce product differentiation, and minimize the cost of distributing goods and services to customers has an important role in increasing

market share and corporate profits. On the other hand, the adoption of policies for the establishment of industries without any knowledge and information, could lead to the loss or reduction of the efficiency of the economic system.

In this paper, one of the weaknesses or shortcomings in the simplification of the location models, which is always the uniformity of the distribution of consumers, is eliminated and replaced by the triangular distribution model. Furthermore, the uncertainty of product quality has been replaced in a range (with three substrates). The results showed that if the difference between the products offered by the two firms is high, then only one firm will provide its products and if this difference is low, then both firms will provide products and two firms will be deployed at one point. If firms 1 and 2 are the only vendors on the market, then they will receive the highest price at the final points. If the indifferent consumer is in the left of the center, then firm 1 receives the highest price at point 0, and if the indifferent consumer is in the right of the center, then firm 1 receives the highest price at point 1. In both cases, firm 2 will receive the highest price at the center. Finally and based on the results of firms, it is suggested that firms choose location and price decisions based on indifferent consumer position and if only one firm is active in the market, it is better to be located on the margin.

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